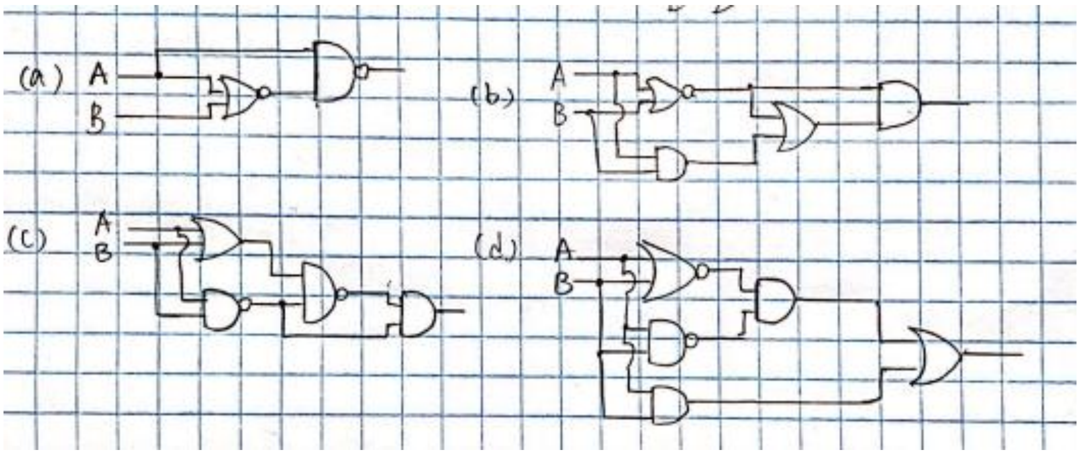
1. Synthesize logic circuits to realize the following functions as written (i.e. no logic simplification), use any of the gates seen in classes:

(a) $\overline{A\cdot\overline{(A+B)}}$, (b) $\left(\overline{A+B}+A\cdot B\right)\cdot\overline{A+B}$

(c) $\overline{(A+B)\cdot\overline{A\cdot B}}\cdot\overline{A\cdot B}$ (d) $\left(\overline{A+B}\right)\cdot\overline{A\cdot B}+A\cdot B.$



2. Use truth tables to prove the following Boolean theorems:

(a) $A\cdot(A+B) = A$, (b) $A+B\cdot C = (A+B)\cdot(A+C)$.

(c) $A+A\cdot B = A$, (d) $A+(B+C) = (A+B)+C$.

Example: (a)

A	B	A+B	A(A+B)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

$A\cdot(A+B) = A$

3. Use Boolean theorems to prove the following identities:

(a) $A\cdot\left(\overline{A}+B\right) = A\cdot B$

(b) $A\cdot\overline{B}\cdot C+A\cdot B\cdot C = A\cdot C$

(c) $(A+B)\left(\overline{A}+C\right)+B\cdot C = A\cdot C+\overline{A}\cdot B$

(d) $(A+C)(A+D)(B+C)(B+D) = A\cdot B+C\cdot D$

3. (a) $A\cdot(\bar{A}+B) = \cancel{A\cdot\bar{A}}^{\,0} + A\cdot B = A\cdot B$

(b) $A\cdot\bar{B}\cdot C + A\cdot B\cdot C = A\cdot C\,\cancel{(\bar{B}+B)}^{\,1} = A\cdot C$

(c) $(A+B)(\bar{A}+C)+B\cdot C = \cancel{A\bar{A}}^{\,0} + \bar{A}B + A\cdot C + \underline{BC+BC}$

$= A\cdot C + \bar{A}\cdot B + BC = A\cdot C + \bar{A}\cdot B$ (Consensus Identity)

(d) $(A+C)(A+D)(B+C)(B+D) = (A+\underline{CD})(B+\underline{CD})$ (#15 Distributive)

$= CD + AB$ (#15 Distributive)

4. Use Boolean algebra to simplify the following expressions:

(a) $\bar{A}B\bar{C}+\bar{A}BC+A\bar{B}\bar{C}+A\bar{B}C$

(b) $\bar{A}\bar{B}\bar{C}\bar{D}+\bar{A}\bar{B}\bar{C}D+\bar{A}\bar{B}C\bar{D}+\bar{A}\bar{B}CD+A\bar{B}\bar{C}\bar{D}+A\bar{B}C\bar{D}$

4. (a) $\bar{A}B\bar{C}+\bar{A}BC+A\bar{B}\bar{C}+A\bar{B}C$

$= \bar{A}B\,\cancel{(\bar{C}+C)}^{\,1} + A\bar{B}\,\cancel{(\bar{C}+C)}^{\,1}$

$= \bar{A}B + A\bar{B} = A \oplus B$

(b) $\bar{A}\bar{B}\bar{C}\bar{D}+\bar{A}\bar{B}\bar{C}D+\bar{A}\bar{B}C\bar{D}+\bar{A}\bar{B}CD+A\bar{B}\bar{C}\bar{D}+A\bar{B}C\bar{D}$

Rule #3 $= \bar{A}\bar{B}\bar{C}\bar{D}+\bar{A}\bar{B}\bar{C}\bar{D}+\bar{A}\bar{B}\bar{C}D+\bar{A}\bar{B}C\bar{D}+\bar{A}\bar{B}C\bar{D}+\bar{A}\bar{B}CD+A\bar{B}\bar{C}\bar{D}+A\bar{B}C\bar{D}$

$= \bar{A}\bar{B}\bar{C}(\bar{D}+D) + \bar{A}\bar{B}\bar{D}(\bar{C}+C) + \bar{A}\bar{B}C(\bar{D}+D) + A\bar{B}\bar{D}(\bar{C}+C)$

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} + \bar{A}\bar{B}C + A\bar{B}\bar{D}$

$= \bar{A}\bar{B}\,\cancel{(\bar{C}+C)}^{\,1} + \bar{B}\bar{D}\,\cancel{(\bar{A}+A)}^{\,1}$

$= \bar{A}\bar{B} + \bar{B}\bar{D}$

CD \ AB	00	01	11	10
00	1			1
01	1			
11	1			
10	1			1

* A K-map can be used to guide your simplification

5. Consider the functions

$$f_1(A,B,C,D) \;=\; \bar{A}BC + (\bar{A}+B+D)(AB\bar{D}+B)$$

(a) Find the complements of the functions above and simplify using De Morgan's laws.

$$\overline{f} = \overline{\overline{A}BC + (\overline{A}+B+D)(AB\overline{D}+B)}$$

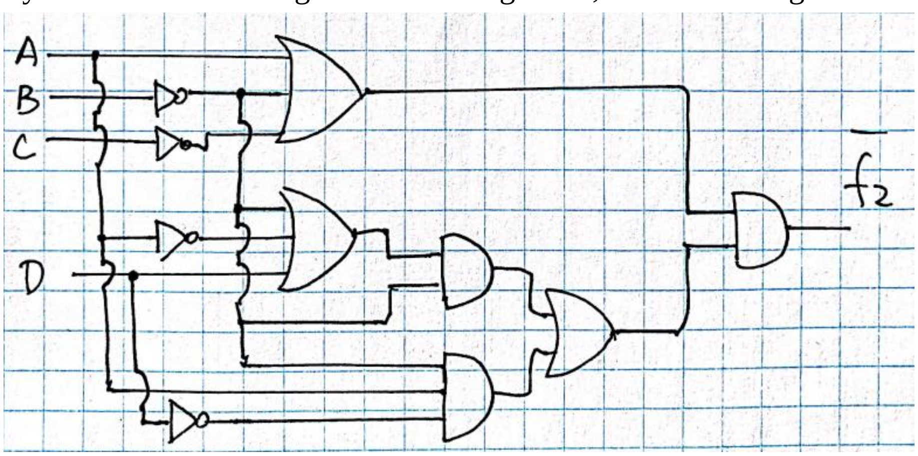
$$\overline{f} = \overline{\overline{A}BC} \cdot \overline{(\overline{A}+B+D)(AB\overline{D}+B)}$$

$$\overline{f} = (A+\overline{B}+\overline{C})\left[\overline{\overline{A}+B+D} + \overline{AB\overline{D}+B}\right]$$

$$\overline{f} = (A+\overline{B}+\overline{C})(A\overline{B}\overline{D} + \overline{AB\overline{D}}\cdot\overline{B})$$

$$\overline{f} = (A+\overline{B}+\overline{C})(A\overline{B}\overline{D} + \overline{B}\cdot(\overline{A}+\overline{B}+D))$$

(b) Synthesize the resulting functions using AND, OR and NOT gates.



6. (a) Write in binary the following decimals: 6_d, 14.625_d, 27_d, 50.3125_d

(b) Write in decimal the following binaries: 00101_b, 101.1_b, 110101_b, 1011.011_b

(a) $6d = 0110b$
$14.625 = 01110.101\,b$
$27d = 011011\,b$
$50.3125d = 0110010.0101\,b$
(b) $00101\,b = 5d$
$101.1\,b = 5.5d$ (if unsigned)
$110101\,b = 65_o = 53d$ (if unsigned)
$1011.011_b = 11.375_d$

7. Express the following decimal numbers using 8-bit signed 2's complement notation:

i) -25_d, ii) 130_d iii) 17_d, iv) -100_d.

i) $-25d = -0001\,1001_b = 1110\,0111_b$ ii) $+130d$ outside the range
iii) $17d = 0001\,0001\,b$ iv) $-100d = -64_h = -0110\,0100_b = 1001\,1100\,b$